

C3 JAN 09

1)  $u = x^2$      $v = (5x-1)^{\frac{1}{2}}$      $\frac{dy}{dx} = vu' + uv'$   
 $u' = 2x$      $v' = \frac{1}{2} \times 5(5x-1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 2x(5x-1)^{\frac{1}{2}} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$$

$$x=2 \quad \frac{dy}{dx} = 4(9)^{\frac{1}{2}} + 10(9)^{-\frac{1}{2}} = 12 + \frac{10}{3} = \frac{46}{3}$$

b)  $u = \sin 2x$      $v = x^2$      $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$   
 $u' = 2\cos 2x$      $v' = 2x$

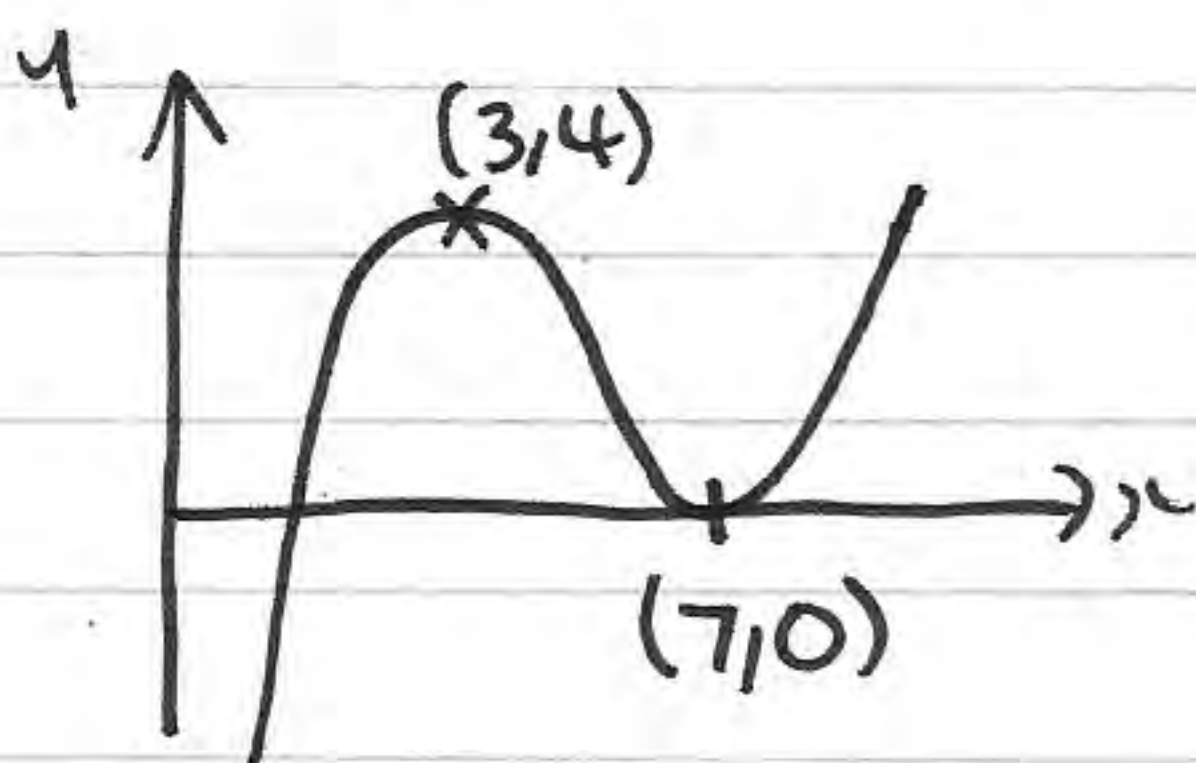
$$\frac{dy}{dx} = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$$

2)  $\frac{2(x+1)}{(x+1)(x-3)} - \frac{(x+1)}{(x-3)} = \frac{2-x-1}{x-3} = \frac{1-x}{x-3}$

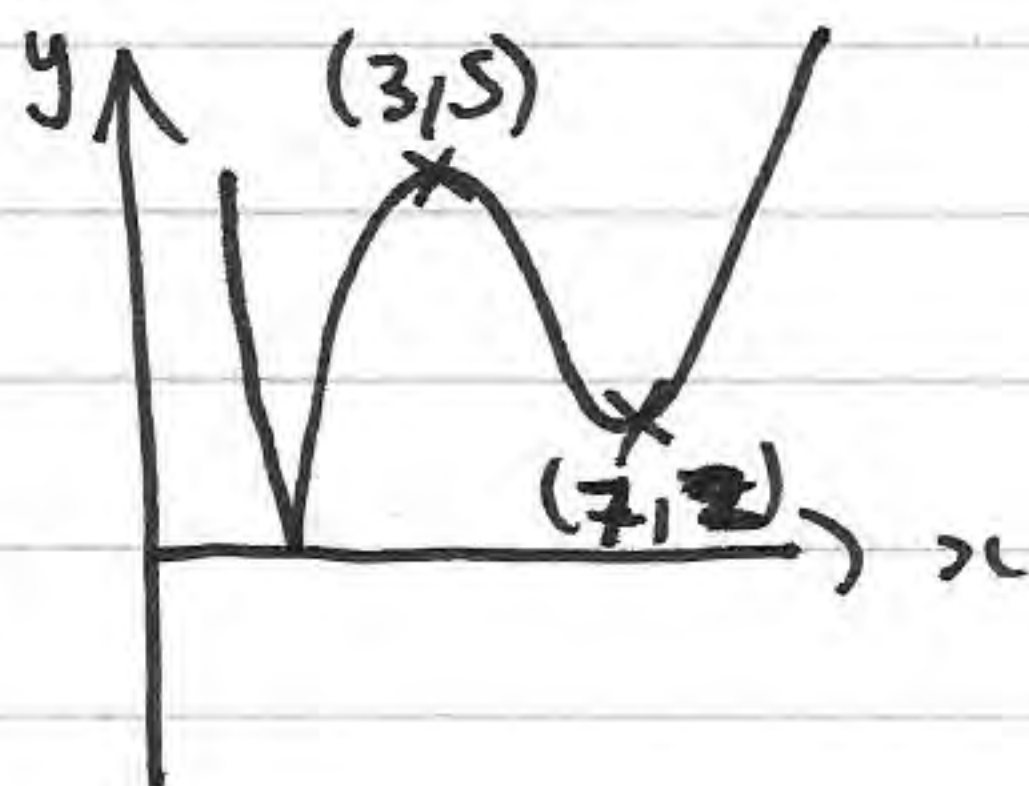
b)  $u = 1-x$      $v = x-3$      $\frac{dy}{dx} = \frac{-(x-3) - (1-x)}{(x-3)^2}$   
 $u' = -1$      $v' = 1$

$$\frac{dy}{dx} = \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2} \neq$$

3)  $y = 2f(x) - 4$   
 $\begin{matrix} \uparrow 2 \\ \downarrow 4 \end{matrix}$   
 $y \times 2 \text{ then } -4$



b)  $|f(x)|$





4)  $x = \cos(2y + \pi) \quad (0, \frac{\pi}{4})$

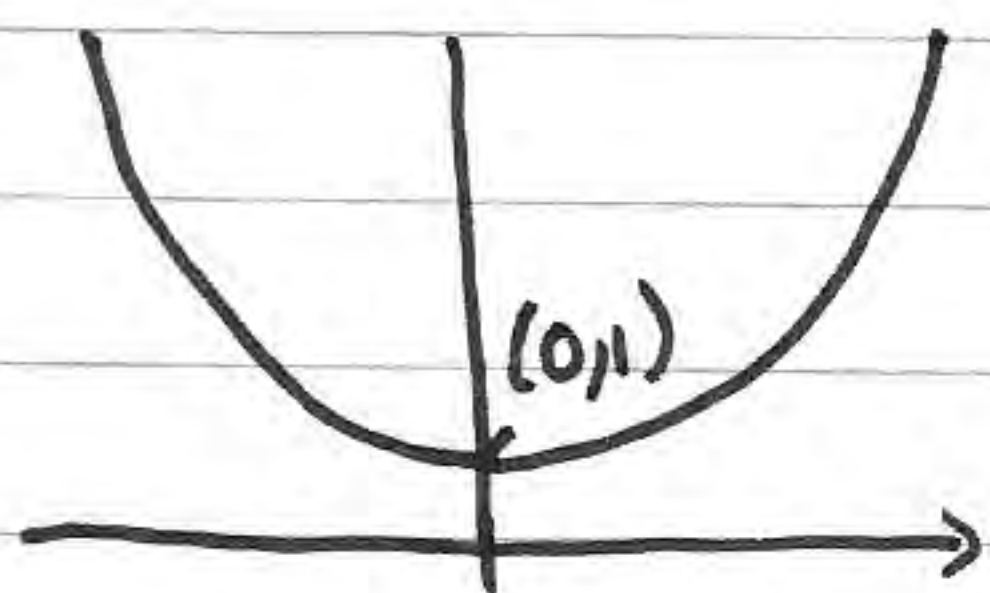
$$\frac{dx}{dy} = -2 \sin(2y + \pi) \Rightarrow \frac{dy}{dx} = \frac{-1}{2 \sin(2y + \pi)}$$

$$y = \frac{\pi}{4} \Rightarrow M_t = \frac{-1}{2 \sin(\frac{\pi}{2} + \pi)} = \frac{-1}{-2} = \frac{1}{2}$$

$$\Rightarrow y - \frac{\pi}{4} = \frac{1}{2}(x - 0) \Rightarrow y = \frac{1}{2}x + \frac{\pi}{4}$$

5)  $g(x) = e^{x^2}$

b)  $fg(x) = f(e^{x^2}) = 3e^{x^2} + \ln e^{x^2}$



range  $y \geq 1$

$$fg(x) = 3e^{x^2} + x^2$$

c)  $x \rightarrow g \rightarrow f \rightarrow fg(x)$

$\geq 0$   
and so  $\geq 1$   
 $\geq 1$

$$fg(x) \geq 3(1) + \ln(1)$$

range  $y \geq 3$

d)  $\frac{d}{dx} [3e^{x^2} + x^2] = 6xe^{x^2} + 2x = x(xe^{x^2} + 2)$

$$\Rightarrow 6xe^{x^2} + 2x = x^2e^{x^2} + 2x$$

$$\Rightarrow (x^2 - 6x)e^{x^2} = 0$$

$$\Rightarrow x(x - 6)e^{x^2} = 0 \quad \underline{x=0}, \underline{x=6}$$

6)  $\sin(3\theta) = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$

$$= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta$$

$$= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$$

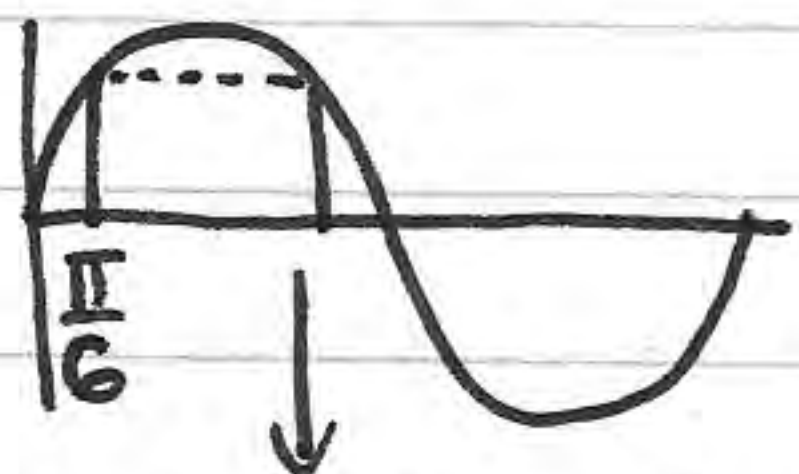
$$= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$$

$$= 3 \sin \theta - 4 \sin^3 \theta \neq$$



$$\text{ii) } 6\sin\theta - 8\sin^3\theta = 1 \Rightarrow 3\sin\theta - 4\sin^3\theta = \frac{1}{2} \Rightarrow \sin 3\theta = \frac{1}{2}$$

$$\Rightarrow 3\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$



$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6} \dots$$

$$+2\pi = \frac{12\pi}{6} \quad +2\pi = \frac{12\pi}{6}$$

$$\textcircled{\div 3} \theta = \frac{\pi}{18}, \frac{5\pi}{18}$$

$$\begin{aligned} \text{b) } \sin 15 &= \sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30 \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\ &= \frac{1}{4}(\sqrt{6} - \sqrt{2}) \quad \# \end{aligned}$$

$$\text{7) } f(x) = 3xe^x - 1$$

$$u = 3x \quad v = e^x$$

$$u' = 3 \quad v' = e^x$$

$$f'(x) = 3e^x + 3xe^x$$

$$\text{TP when } f'(x) = 0 \Rightarrow 3e^x(1+x) = 0$$

$$\Rightarrow \underline{x = -1}$$

$$P\left(-1, -\frac{3}{e} - 1\right)$$

$$\text{b) } x_0 = 0.25 \quad x_1 = 0.2596 \quad x_2 = 0.2571 \quad x_3 = 0.2578$$

$$\begin{aligned} \text{c) } f(0.25765) &= 0.000109 > 0 && \text{Change of sign} \\ f(0.25755) &= -0.00038 < 0 && \Rightarrow \underline{\alpha \in 0.2576} \text{ (4dp)} \end{aligned}$$



$$8) R \cos(\theta - \alpha) = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

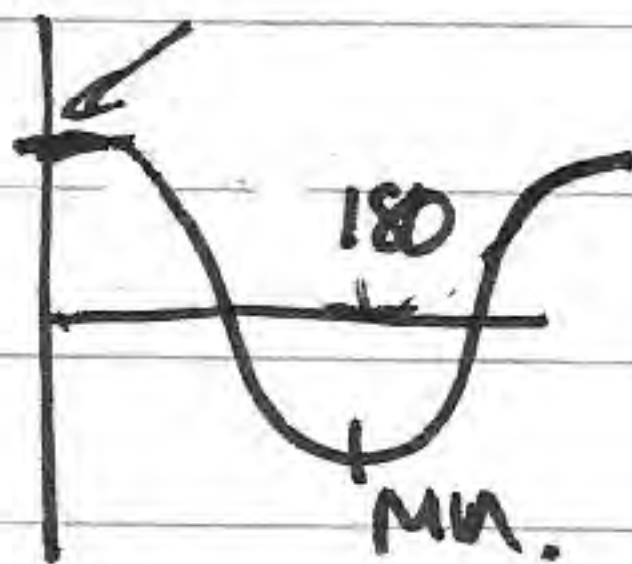
$$3 \cos \theta + 4 \sin \theta$$

$$\frac{R \sin \alpha = 4}{R \cos \alpha = 3} \Rightarrow \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$$

$$R^2 = 3^2 + 4^2 \Rightarrow R = 5$$

$$5 \cos(\theta - 53.1^\circ)$$

b) max = 5



max occurs when  $\theta - 53.1 = 0$   
 $\Rightarrow \theta = \underline{53.1}$

c)  $f(t) = 3 \cos(15t) + 4 \sin(15t) + 10 \Rightarrow \text{Min } f(t) = 10 - 5 = \underline{5}$

min = -5

min when  $\theta - 53.1 = 180$

$\Rightarrow \theta = 233.1$

$15t = 233.1 \Rightarrow t = \underline{15.54}$

(0332)